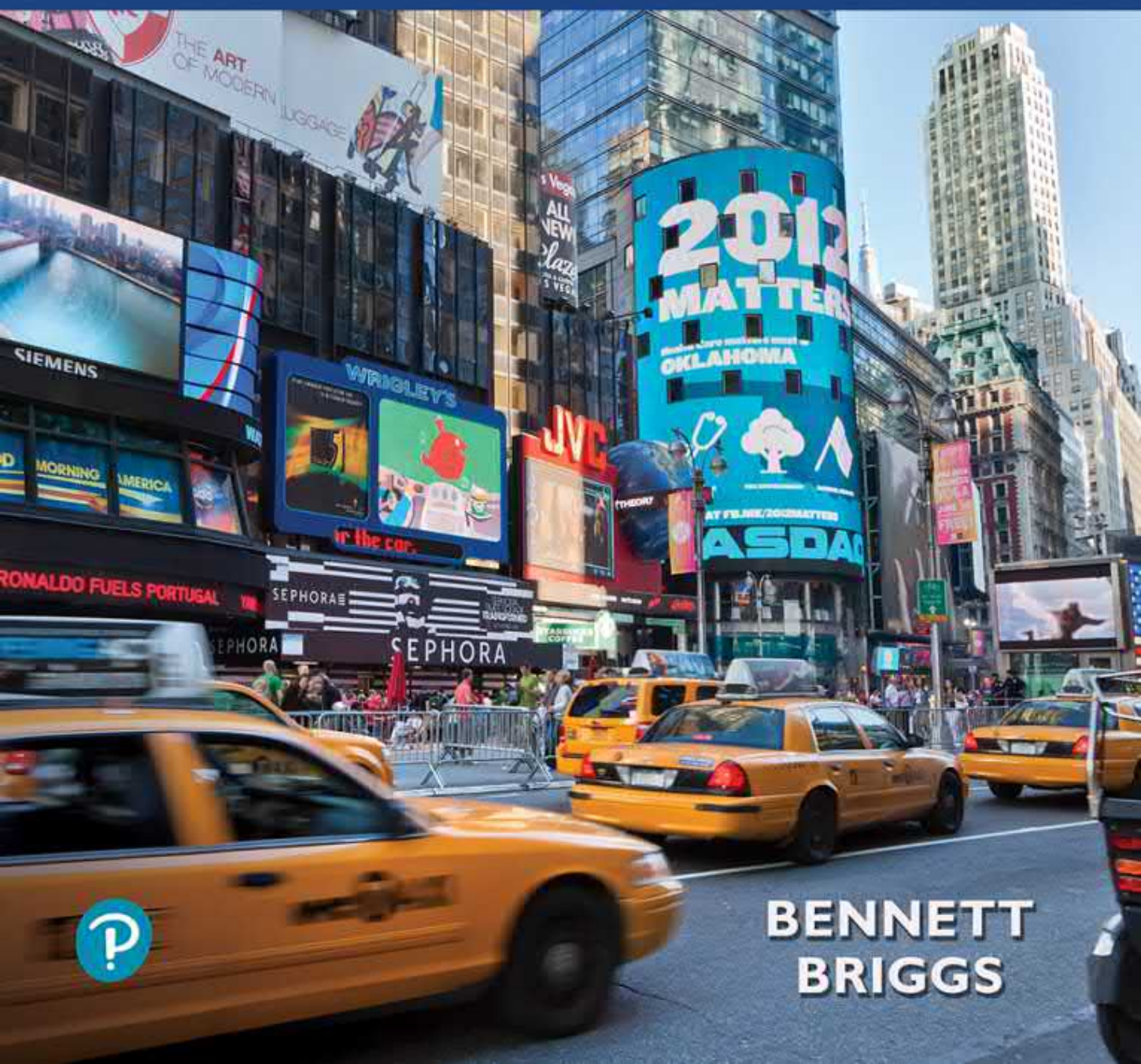


USING & UNDERSTANDING MATHEMATICS

A QUANTITATIVE REASONING APPROACH

SEVENTH EDITION



**BENNETT
BRIGGS**

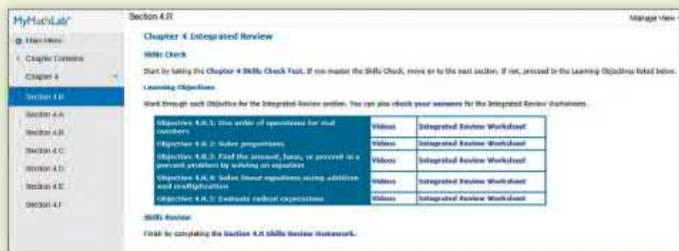


MyLab Math Online Course for *Using & Understanding Mathematics*, 7th edition, by Bennett and Briggs



MyLab Math with Integrated Review

In addition to the standard MyLab™ Math course, *Using & Understanding Mathematics* offers a **MyLab Math course with Integrated Review**, giving students the support they need before completing their homework. A Skills Check is available with each Integrated Review module to ensure that students are prepared for the upcoming chapter material.



Cell Phones and Driving

Use this activity to gain a sense of the kinds of problems this chapter will enable you to analyze. Additional activities are available online in MyLab Math.

Is it safe to use a cell phone while driving? The science of statistics provides a way to approach this question, and the results of many studies indicate that the answer is no. The National Safety Council estimates that approximately 1.5 million car crashes each year (more than a quarter of the total) are caused by some type of distraction, most commonly the use of a cell phone for talking or texting. In fact, some studies suggest that merely talking on a cell phone makes you as dangerous as a drunk driver. As preparation for your study of statistics in this chapter, work individually or in groups to research the issues raised in the following questions. Discuss your findings.



Integrated Activities

Activities are heavily integrated into this edition of *Using & Understanding Mathematics*. Each chapter opens with an **Activity** designed to spur discussion of some interesting facet of the topics covered in the chapter.

NEW! Activity Manual with Integrated Review Worksheets

The **Activity Manual** provides additional activities for each chapter in a workbook format. In addition to the activities, the workbook includes Integrated Review Worksheets to support the Integrated Review version of the MyLab Math course.

Math for College, Career, and Life

We all use math in our day-to-day lives. The goal of this book is to improve students' mathematical literacy so that they can use math more effectively in everyday life. Mathematics can help students better understand a variety of topics and issues, making them more aware of the uses and abuses of numbers. The ultimate goal is to help them become better educated citizens who are successful in their college experiences, their careers, and their lives.

In Your World boxes focus on topics that students are likely to encounter in the world around them—in the news, in consumer decisions, or in political discussions. The connection to the real world is further enhanced by **In Your World** exercises, designed to spur additional research or discussion that will help students relate the unit's topics to the themes of college, career, and life.


IN YOUR WORLD
Fact Checking on the Web

While the Web is often a source of false, inaccurate, or biased information, it is also a great source for checking the accuracy of information. A good way to start is with "fact checking" websites, as long as you also verify that the fact checkers have a reputation for fairness and accuracy. Here are a few reputable fact-checking sites:

- For political fact checking: **FactCheck.org**, supported by the non-partisan and nonprofit Annenberg Public Policy Center; **PolitiFact.com**, a Pulitzer Prize-winning site from the *Orange Bay Times*; and **"The Fact Checker,"** a blog hosted on the *Washington Post's* website.
- For rumors, urban myths, and other odd claims, **Scoops.com** has a solid reputation for accuracy.

To check the validity of messages you receive by email, try **SniffOutSpam.com**, which also has a strong reputation for fairness and accuracy.

If none of these sources has covered the claim you are investigating, try a plain language Web search. For example, if you type the first sentence of the *Mars* claim ("On August 27, Mars will look as large and bright as the full Moon...") into a search engine, you'll get dozens of hits that discuss the claim and why it is false. Of course, if your search turns up conflicting claims about accuracy, you'll still need to decide which claims to believe.



IN YOUR WORLD

45. **Political Action.** This unit outlined numerous budgetary problems facing the U.S. government, as they stood at the time the text was written in 2017. Has there been any significant political action to deal with any of these problems? Learn what, if anything, has changed since 2017; then write a one-page position paper outlining your own recommendations for the future.

46. **Debt Problem.** How serious a problem is the gross debt? Find arguments on both sides of this question. Summarize the arguments, and state your own opinion.

Concepts are brought to life in five **In Your World** videos, assignable within MyLab Math. These videos provide further connections to students' everyday lives in an entertaining and engaging way.



NEW! Integrated StatCrunch

StatCrunch, powerful web-based statistical software, allows students to harness technology to perform data analysis. This edition includes StatCrunch exercises in relevant exercise sets and some questions assignable in MyLab Math.

Why Should You Care About Quantitative Reasoning?

Quantitative reasoning is the ability to interpret and reason with information that involves numbers or mathematical ideas. It is a crucial aspect of literacy, and it is essential in making important decisions and understanding contemporary issues.

The topics covered in this text will help you work with quantitative information and make critical decisions. For example:

- You should possess strong skills in critical and logical thinking so that you can make wise personal decisions, navigate the media, and be an informed citizen. For example, do you know why you'd end up behind if you accepted a temporary 10% pay cut now and then received a 10% pay raise later? This particular question is covered in Unit 3A, but throughout the book you'll learn how to evaluate quantitative questions on topics ranging from personal decisions to major global issues.
- You should have a strong number sense and be proficient at estimation so that you can put numbers from the news into a context that makes them understandable. For example, do you know how to make sense of the more than \$20 trillion federal debt? Unit 3B discusses how you can put such huge numbers in perspective, and Unit 4F discusses how the federal debt grew so large.
- You should possess the mathematical tools needed to make basic financial decisions. For example, do you enjoy a latte every morning before class? Sometimes two? Unit 4A explores how such a seemingly harmless habit can drain more than \$2400 from your wallet every year.
- You should be able to read news reports of statistical studies in a way that will allow you to evaluate them critically and decide whether and how they should affect your personal beliefs. For example, how should you decide whether a new opinion poll accurately reflects the views of Americans? Chapter 5 covers the basic concepts that lie behind the statistical studies and graphics you'll see in the news, and discusses how you can decide for yourself whether you should believe a statistical study.
- You should be familiar with basic ideas of probability and risk and be aware of how they affect your life. For example, would you pay \$30,000 for a product that, over 20 years, will kill nearly as many people as live in San Francisco? In Unit 7D, you'll see that the answer is very likely yes—just one of many surprises that you'll encounter as you study probability in Chapter 7.
- You should understand how mathematics helps us study important social issues, such as global warming, the growth of populations, the depletion of resources, apportionment of congressional representatives, and methods of voting. For example, Unit 12D discusses the nature of redistricting and how gerrymandering has made congressional elections less competitive than they might otherwise be.

In sum, this text will focus on understanding and interpreting mathematical topics to help you develop the quantitative reasoning skills you will need for college, career, and life.

7th EDITION

Using & Understanding **MATHEMATICS**

A Quantitative Reasoning Approach

Jeffrey Bennett

University of Colorado at Boulder

William Briggs

University of Colorado at Denver



330 Hudson Street, NY NY 10013

Director, Portfolio Management: Anne Kelly
Courseware Portfolio Specialist: Marnie Greenhut
Courseware Portfolio Management Assistant: Stacey Miller
Content Producer: Patty Bergin
Managing Producer: Karen Wernholm
TestGen Content Manager: Mary Durnwald
Manager, Content Development (Math XL): Robert Carroll
Media Producer: Nick Sweeny
Product Marketing Manager: Kyle DiGiannantonio
Field Marketing Manager: Andrew Noble
Marketing Assistant: Hanna Lafferty
Senior Author Support/Technology Specialist: Joe Vetere
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This book is dedicated to everyone who wants a better understanding of our world, and especially to those who have struggled with mathematics in the past. We hope this book will help you achieve your goals.

And it is dedicated to those who make our own lives brighter, especially Lisa, Julie, Katie, Grant, and Brooke.

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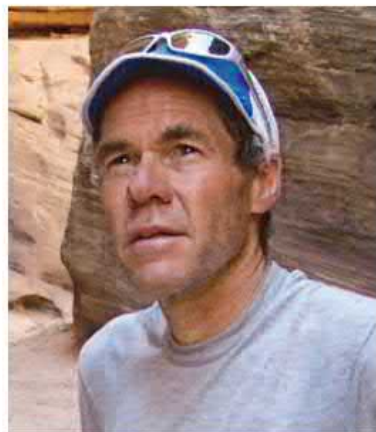
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ABOUT THE AUTHORS



Jeffrey Bennett served as the first director of the program “Quantitative Reasoning and Mathematical Skills” at the University of Colorado at Boulder, where he developed the groundbreaking curriculum that became the basis of this textbook. He holds a BA in biophysics (University of California, San Diego) and an MS and a PhD in astrophysics (University of Colorado), and has focused his career on math and science education. In addition to co-authoring this textbook, he is also the lead author of best-selling college textbooks on statistical reasoning, astronomy, and astrobiology, and of more than a dozen books for children and adults. All six of his children’s books have been selected for NASA’s “Story Time From Space” (storytimefromspace.com), a project in which astronauts on the International Space Station read books aloud and videos are posted that anyone in the world can watch for free. His most recent books include *I, Humanity* for children and *Math for Life* and *A Global Warming Primer* for the general public. Among his many other endeavors, Dr. Bennett proposed and coled the development of the Voyage Scale Model Solar System, which is located outside the National Air and Space Museum on the National Mall in Washington, DC. Learn more about Dr. Bennett and his work at www.jeffreybennett.com.



William Briggs was on the mathematics faculty at Clarkson University for 6 years and at the University of Colorado at Denver for 23 years, where he taught both undergraduate and graduate courses, with a special interest in applied mathematics. During much of that time, he designed and taught courses in quantitative reasoning. In addition to this book, he has co-authored textbooks on statistical reasoning and calculus, as well as monographs in computational mathematics. He recently completed the book *How America Got Its Guns* (University of New Mexico Press). Dr. Briggs is a University of Colorado President’s Teaching Scholar and the recipient of a Fulbright Fellowship to Ireland; he holds a BA degree from the University of Colorado and an MS and a PhD from Harvard University.

PREFACE

Human history
becomes more and more
a race between education
and catastrophe.

—H. G. Wells
The Outline of History,
1920

To the Student

There is no escaping the importance of mathematics in the modern world. However, for most people, the importance of mathematics lies not in its abstract ideas, but in its application to personal and social issues. This textbook is designed with such practical considerations in mind. In particular, this book has three specific purposes:

- to prepare you for the mathematics you will encounter in other college courses, particularly core courses in social and natural sciences;
- to develop your ability to reason with quantitative information in a way that will help you achieve success in your career; and
- to provide you with the critical thinking and quantitative reasoning skills you need to understand major issues in your life.

We hope this book will be useful to everyone, but it is designed primarily for those who are *not* planning to major in a field that requires advanced mathematical skills. In particular, if you've ever felt any fear or anxiety about mathematics, we've written this book with you in mind. We hope that, through this book, you will discover that mathematics is much more important and relevant to your life than you thought and not as difficult as you previously imagined.

Whatever your interests—social sciences, environmental issues, politics, business and economics, art and music, or any of many other topics—you will find many relevant and up-to-date examples in this book. But the most important idea to take away from this book is that mathematics can help you understand a variety of topics and issues, making you a more aware and better educated citizen. Once you have completed your study of this book, you should be prepared to understand most quantitative issues that you will encounter.

To the Instructor

Whether you've taught this course many times or are teaching it for the first time, you are undoubtedly aware that mathematics courses for non-majors present challenges that differ from those presented by more traditional courses. First and foremost, there isn't even a clear consensus on what exactly should be taught in these courses. While there's little debate

about what mathematical content is necessary for science, technology, engineering, and mathematics (STEM) students—for example, these students all need to learn algebra and calculus—there's great debate about what we should teach non-STEM students, especially the large majority who will *not* make use of formal mathematics in their careers or daily lives.

As a result of this debate, core mathematics courses for non-STEM students represent a broad and diverse range. Some schools require these students to take a traditional, calculus-track course, such as college algebra. Others have instituted courses focused on some of the hidden ways in which contemporary mathematics contributes to society, and still others have developed courses devoted almost exclusively to financial literacy. Each of the different course types has its merits, but we believe there is a better option, largely because of the following fact: The vast majority (typically 95%) of non-STEM students will *never* take another college mathematics course after completing their core requirement.

Given this fact, we believe it is essential to teach these students the mathematical ideas that they will *need* for their remaining college course work, their careers, and their daily lives. In other words, we must emphasize those topics that are truly important to the future success of these students, and we must cover a broad range of such topics. The focus of this approach is less on formal calculation—though some is certainly required—and more on teaching students how to think critically with numerical or mathematical information. In the terminology adopted by MAA, AMATYC, and other mathematical organizations, students need to learn *quantitative reasoning* and to become *quantitatively literate*. There's been a recent rise in the popularity of quantitative reasoning courses for non-STEM students. This book has been integral to the quantitative reasoning movement for years and continues to be at the forefront as an established resource designed to help you succeed in teaching quantitative reasoning to your students.

The Key to Success: A Context-Driven Approach

Broadly speaking, approaches to teaching mathematics can be divided into two categories:

- A *content-driven approach* is organized by mathematical ideas. After each mathematical topic is presented, examples of its applications are shown.

- A *context-driven approach* is organized by practical contexts. Applications drive the course, and mathematical ideas are presented as needed to support the applications.

The same content can be covered through either approach, but the context-driven approach has an enormous advantage: It motivates students by showing them directly how relevant mathematics is to their lives. In contrast, the content-driven approach tends to come across as “learn this content because it’s good for you,” causing many students to tune out before reaching the practical applications. For more details, see our article “General Education Mathematics: New Approaches for a New Millennium” (*AMATYC Review*, Fall 1999) or the discussion in the Epilogue of the book *Math for Life* by Jeffrey Bennett (Big Kid Science, 2014).

The Challenge: Winning Over Your Students

Perhaps the greatest challenge in teaching mathematics lies in winning students over—that is, convincing them that you have something useful to teach them. This challenge arises because by the time they reach college, many students dislike or fear mathematics. Indeed, the vast majority of students in general education mathematics courses are there not by choice, but because such courses are required for graduation. Reaching your students therefore requires that you teach with enthusiasm and convince them that mathematics is useful and enjoyable.

We’ve built this book around two important strategies that are designed to help you win students over:

- Confront negative attitudes about mathematics head on, showing students that their fear or loathing is ungrounded and that mathematics is relevant to their lives. This strategy is embodied in the Prologue of this book (pages P1–P13), which we urge you to emphasize in class. It continues implicitly throughout the rest of the text.
- Focus on goals that are meaningful to students—namely, on the goals of learning mathematics for *college*, *career*, and *life*. Your students will then learn mathematics because they will see how it affects their lives. This strategy forms the backbone of this book, as we have tried to build every unit around topics relevant to college, career, and life.

Modular Structure of the Book

Although we have written this book so that it can be read as a narrative from beginning to end, we recognize that many instructors might wish to teach material in a different order than we have chosen or to cover only selected portions of the text, as time allows, for classes of different length or for

students at different levels. We have therefore organized the book with a modular structure that allows instructors to create a customized course. The 12 chapters are organized broadly by contextual areas. Each chapter, in turn, is divided into a set of self-contained *units* that focus on particular concepts or applications. In most cases, you can cover chapters in any order or skip units that are lower priority for your particular course. The following outline describes the flow of each chapter:

Chapter Overview Each chapter begins with a two-page overview consisting of an introductory paragraph and a multiple-choice question designed to illustrate an important way in which the chapter content connects with the book themes of *college*, *career*, and *life*. The overview also includes a motivational quote and a unit-by-unit listing of key content; the latter is designed to show students how the chapter is organized and to help instructors decide which units to cover in class.

Chapter Activity Each chapter next offers an activity designed to spur student discussion of some interesting facet of the topics covered in the chapter. The activities may be done either individually or in small groups. A new Activity Manual containing additional activities is available with this seventh edition in print form and also in MyLab Math.

Numbered Units Each chapter consists of numbered units (e.g., Unit 1A, Unit 1B, ...). Each unit begins with a short introduction and includes the following key features:

- **Headings to Identify Key Topics.** In keeping with the modularity, each subtopic within a unit is clearly identified so that students understand what they will be learning.
- **Summary Boxes.** Key definitions and concepts are highlighted in summary boxes for easy reference.
- **Examples and Case Studies.** Numbered examples are designed to build understanding and to offer practice with the types of questions that appear in the exercises. Each example is accompanied by a “Now try ...” suggestion that relates the example to specific similar exercises. Occasional case studies go into more depth than the numbered examples.
- **Exercises.** Each unit concludes with a set of exercises, subdivided into the following categories:
 - **Quick Quiz.** This ten-question quiz appears at the end of each unit and allows students to check whether they understand key concepts before starting the exercise set. Note that students are asked not only to choose the correct multiple-choice answer but also to write a brief explanation of the reasoning behind their choice. Answers are included in the back of the text.

- **Review Questions.** Designed primarily for self-study, these questions ask students to summarize the important ideas covered in the unit and generally can be answered simply by reviewing the text.
- **Does It Make Sense?** These questions ask students to determine whether a short statement makes sense, and explain why or why not. These exercises are generally easy once students understand a particular concept, but difficult otherwise; they are therefore an excellent probe of comprehension.
- **Basic Skills & Concepts.** These questions offer practice with the concepts covered in the unit. They can be used for homework assignments or for self-study (answers to most odd-numbered exercises appear in the back of the book). These questions are referenced by the “Now try ...” suggestions in the unit.
- **Further Applications.** Through additional applications, these exercises extend the ideas and techniques covered in the unit.
- **In Your World.** These questions are designed to spur additional research or discussion that will help students relate the unit content to the book themes of college, career, and life.
- **Technology Exercises.** For units that include one or more Using Technology features, these exercises give students an opportunity to practice calculator or software skills that have been introduced. Some of these exercises are designed to be completed with StatCrunch (www.statcrunch.com), which comes with the MyLab Math course. Applications using StatCrunch, powerful Web-based statistical software that allows users to collect data, perform analyses, and generate compelling results, are included in this edition for the first time.

Chapter Summary Appearing at the end of each chapter, the Chapter Summary offers a brief outline of the chapter’s content, including page numbers, that students can use as a study guide.

Additional Pedagogical Features In addition to the standard features of all chapters listed above, several other pedagogical features occur throughout the text:

- **Think About It.** These features pose short conceptual questions designed to help students reflect on important new ideas. They also serve as excellent starting points for classroom discussions and, in some cases, can be used as a basis for clicker questions.
- **Brief Review.** This feature appears when a key mathematical skill is first needed; topics include fractions, powers and roots, basic algebraic operations, and more. The word “review” indicates that most

students will have learned these skills previously, but many will need review and practice. Practice is available in the exercise sets, with relevant exercises identified by a “Now try ...” suggestion at the end of the Brief Review.

- **In Your World.** These features focus on topics that students are likely to encounter in the world around them, whether in the news, in consumer decisions, or in political discussions. Examples include how to understand jewelry purchases, how to invest money in a sensible way, and how to evaluate the reliability of pre-election polls. (*Note:* These features are not necessarily connected directly to the In Your World exercises, but both have direct relevance to students’ world.)
- **Using Technology.** These features give students clear instructions in the use of various technologies for computation, including scientific calculators, Microsoft Excel, and online technologies such as those built into Google. Book-specific TI Tech Tips containing instructions for performing computations with a graphing calculator, such as the TI-83 or TI-84, are available in the Tools for Success section of MyLab Math.
- **Caution!** New to the seventh edition, these short notes, integrated into examples or text, highlight common errors that students should be careful to avoid.
- **Mathematical Insight.** This feature, which occurs less frequently than the others, builds on mathematical ideas in the main narrative but goes somewhat beyond the level of other material in the book. Examples of the topics covered are proof of the Pythagorean theorem, Zeno’s paradox, and derivations of the financial formulas used for savings plans and mortgage loans.
- **Margin Features.** The margins contain several types of short features: **By the Way**, which offers interesting notes and asides relevant to the topic at hand; **Historical Note**, which gives historical context to the topic at hand; and **Technical Note**, which offers details that are important mathematically, but generally do not affect students’ understanding of the material. The margins also contain occasional quotations.

Prerequisite Mathematical Background

Because of its modular structure and the inclusion of the Brief Review features, this book can be used by students with a wide range of mathematical backgrounds. Many of the units require nothing more than arithmetic and a willingness to think about quantitative issues in new ways. Only a few units use techniques of algebra or geometry, and those skills are reviewed as they arise. This book should therefore be accessible to any student who has completed two or more years of high school math-

ematics. However, *this book is not remedial*: Although much of the book relies on mathematical techniques from secondary school, the techniques arise in applications that students generally are not taught in high school and that require students to demonstrate their critical thinking skills.

For courses in which students do require more extensive prerequisite review, we have created a version of the *Using & Understanding Mathematics* MyLab Math course called *Using & Understanding Mathematics with Integrated Review* that includes just-in-time review of selected prerequisite topics.

Note on “Developmental Math” We are often asked whether this text can be used by students for whom placement tests suggest that they belong in developmental mathematics courses. In most cases, we believe the answer to be a resounding “yes.” Our experience suggests that many students who do poorly on mathematics placement tests are not really as weak as these tests may suggest. Most students *did* learn basic mathematical skills at one time, and if the skills arise with context (as they do in this book), we’ve found that students can quickly relearn them. This is especially true if you provide the students with a little bit of extra practice as offered in our Brief Review features or by the resources in MyLab Math or MyLab Math with Integrated Review. Indeed, we believe that most students in this situation will learn basic mathematical skills *better* by taking a quantitative reasoning course based on this textbook than they will by taking a developmental course.

Changes in the Seventh Edition

We’ve been pleased by the positive responses from so many users of previous editions of this text. Nevertheless, a book that relies heavily on facts and data always requires a major updating effort to keep it current, and we are always looking for ways to improve clarity and pedagogy. As a result, users of prior editions will find many sections of this book to have been substantially revised or rewritten. The changes are too many to list here, but some of the more significant changes are the following.

Chapter 1 We significantly revised Units 1A and 1E with the particular goal of helping students evaluate media information and recognize “fake news.”

Chapter 2 We reorganized and significantly rewrote this entire chapter to introduce a basic problem-solving strategy in Unit 2A. Moreover, we modified the four-step strategy

presented in previous editions to create a simpler three-step strategy called “Understand-Solve-Explain.” We have found that this strategy is easier for students to remember and therefore easier for them to put into practice.

Chapters 3 and 4 These two chapters contain several units that revolve around economic data such as demographic data, the Consumer Price Index, interest rates, taxes, and the federal budget. These data obviously required major updates given the changes that have occurred in the U.S. economy in the four years since the last edition. In addition, we’ve added basic ideas about health insurance to our discussion of personal finances in Unit 4A.

Chapters 5 and 6 These chapters focus on statistical data, which means we updated or replaced large sections of the chapter content to include more current data.

Chapter 7 We significantly revised Section 7D on risk, both for greater clarity and to update data.

Chapters 8 and 9 Units 8B, 8C, and 9C all rely heavily on population data, which means we revised significant portions of these units to reflect the latest global demographic data.

Chapter 12 The 2016 election provided numerous new examples for our discussion of the electoral college in Unit 12A. Other recent examples of the intersection of mathematics and politics also provide interesting new examples and exercises throughout this chapter.

In Your World We’ve added seven new In Your World features, so every chapter now has at least one, further showcasing math for college, career, and life.

Caution! These short notes highlighting common errors are new to this edition.

Exercise Sets We’ve thoroughly revised the exercise sets: Over 30% of the exercises are changed or new.

StatCrunch StatCrunch has been newly integrated into the MyLab Math course and relevant Technology Exercises.

Video Program The seventh edition is accompanied by an all-new video program consisting of both familiar lecture-style videos for every example and innovative concept videos.

Resources for Success

MyLab Math Online Course for *Using & Understanding Mathematics: A Quantitative Reasoning Approach*, 7th edition

by Jeffrey Bennett and William Briggs

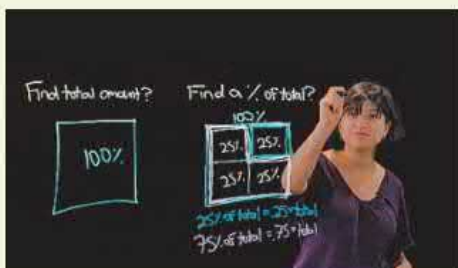
MyLab™ Math is available to accompany Pearson's market-leading text offerings. To give students a consistent tone, voice, and teaching method, each text's flavor and approach are tightly integrated throughout the accompanying MyLab Math course, making learning the material as seamless as possible.

NEW! Lecture Videos

Brand-new lecture videos for every example are fresh and modern and are accompanied by assessment questions that give the instructor the ability to not just assign the videos but gauge student understanding.

Simple Interest Account		
End of Year	Interest Paid	Old Balance + Interest = New Balance
1	$10\% \times \$200 = \20	$\$200 + \$20 = \$220$
2	$10\% \times \$200 = \20	$\$220 + \$20 = \$240$
3	$10\% \times \$200 = \20	$\$240 + \$20 = \$260$
4	$10\% \times \$200 = \20	$\$260 + \$20 = \$280$
5	$10\% \times \$200 = \20	$\$280 + \$20 = \$300$

At the end of 5 years, you have earned \$100 in simple interest and have a total of \$300 invested in the account.



NEW! Concept Videos

Dynamic lightboard videos focus on some of the most interesting and challenging concepts so students can better grasp them. Exciting visuals are used to explain concepts such as comparisons of quantities, student loans, and percentages in the world around us.

NEW! StatCrunch Integration

StatCrunch is powerful web-based statistical software that allows users to collect data, perform analyses, and generate compelling results. For this seventh edition, StatCrunch questions have been added to relevant Technology Exercises and access to the software has been integrated into the MyLab Math course.



Resources for Success

Instructor Resources

MyLab Math with Integrated Review

This MyLab Math course option can be used in co-requisite courses, or simply to help students who enter the quantitative reasoning course lacking prerequisite skills or a full understanding of prerequisite concepts.

- For relevant chapters, students begin with a Skills Check assignment to pinpoint which prerequisite developmental topics, if any, they need to review.
- Those who require additional review proceed to a personalized homework assignment that focuses on the specific prerequisite topics on which they need remediation.
- Students can also review the relevant prerequisite concepts using videos and Integrated Review Worksheets in MyLab Math. The Integrated Review Worksheets are also available in printed form as part of the Activity Manual with Integrated Review Worksheets.

Specific to the *Using & Understanding Mathematics* MyLab Math course:

- NEW! Completely new lecture video program with corresponding assessment
- NEW! Dynamic concept videos
- NEW! Interactive concept videos with corresponding assessment
- NEW! Animations with corresponding assessment
- NEW! Integration of StatCrunch in the left-hand navigation of the MyLab Math course makes it easy to access the software for completion of the Technology Exercises that use StatCrunch.
- Bonus unit on mathematics and business, including assessment

Instructor's Edition

(ISBNs: 0-13-470522-X / 978-0-13-470522-4)

The Instructor's Edition of the text includes answers to all of the exercises and Quick Quizzes in the back of the book.

The following resources are **ONLINE ONLY** and are available for download from the Pearson Higher Education catalog at www.pearson.com/us/sign-in.html or within your MyLab Math course.

Instructor's Solution Manual

James Lapp

This manual includes answers to all of the text's Think About It features, Quick Quizzes, Review Questions, and Does It Make Sense? questions and detailed, worked-out solutions to all of the Basic Skills & Concepts, Further Applications, and Technology Exercises (including StatCrunch exercises).

Instructor's Testing Manual

Dawn Dabney

The Testing Manual provides four alternative tests per chapter, including answer keys.

TestGen

TestGen[®] (www.pearsoned.com/testgen) enables instructors to build, edit, print, and administer tests using a computerized bank of questions developed to cover all the objectives of the text. TestGen is algorithmically based, allowing instructors to create multiple but equivalent versions of the same question or test with the click of a button. Instructors can also modify test bank questions or add new questions. The software and test bank can be downloaded from Pearson's Instructor Resource Center.

PowerPoint Lecture Presentation

These editable slides present key concepts and definitions from the text. Instructors can add art from the text located in the Image Resource Library in MyLab Math or add slides they have created. PowerPoint slides are fully accessible.

Image Resource Library

This resource in the MyLab Math course contains all the art from the text for instructors to use in their own presentations and handouts.

Student Resources

Student's Study Guide and Solutions Manual

(ISBNs: 0-13-470524-6 / 978-0-13-470524-8)

James Lapp

This manual contains answers to all Quick Quiz questions and to odd-numbered Review Questions and Does It Make Sense? questions, as well as worked-out solutions to odd-numbered Basic Skills & Concepts, Further Applications, and Technology Exercises (including StatCrunch exercises).

NEW! Activity Manual with Integrated Review Worksheets

(ISBNs: 0-13-477664-X / 978-0-13-477664-4)

Compiled by Donna Kirk, The College of St. Scholastica

More than 30 activities correlated to the textbook give students hands-on experiences that reinforce the course content. Activities can be completed individually or in a group. Each activity includes an overview, estimated time of completion, objectives, guidelines for group size, and list of materials needed. Additionally, the manual provides the worksheets for the Integrated Review version of the MyLab Math course.

Acknowledgments

A textbook may carry its authors' names, but it is the result of hard work by hundreds of committed individuals. This book has been under development for more than 30 years, and even its beginnings were a group effort, as one of the authors was a member of a committee at the University of Colorado that worked to establish one of the nation's first courses in quantitative reasoning. Since that beginning, the book has benefited from input and feedback from many faculty members and students.

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Prologue

LITERACY FOR THE MODERN WORLD

Equations are just the boring part of mathematics.

—Stephen Hawking, physicist



If you're like most students enrolled in a course using this text, you may think that your interests have relatively little to do with mathematics. But as you will see, nearly every career today requires the use and understanding of some mathematics. Furthermore, the ability to reason quantitatively is crucial for the decisions that we face daily as citizens in a modern technological society. In this Prologue, we'll discuss why mathematics is so important, why you may be better at it than you think, and how this course can provide you with the quantitative skills needed for your college courses, your career, and your life.

Q

Imagine that you're at a party and you've just struck up a conversation with a dynamic, successful lawyer. Which of the following are you most likely to hear her say during your conversation?

- A "I really don't know how to read very well."
- B "I can't write a grammatically correct sentence."
- C "I'm awful at dealing with people."
- D "I've never been able to think logically."
- E "I'm bad at math."

A

We all know that the answer is E, because we've heard it so many times. Not just from lawyers, but from businesspeople, actors and athletes, construction workers and sales clerks, and sometimes even teachers and CEOs. It would be difficult to imagine these same people admitting to any of choices A through D, but many people consider it socially acceptable to say that they are "bad at math." Unfortunately, this social acceptability comes with some very negative social consequences. (See the discussion about Misconception Seven on page P-7.)



Job Satisfaction

Each chapter in this textbook begins with an activity, which you may do individually or in groups. For this Prologue, the opening activity will help you examine the role of mathematics in careers. Additional activities are available online in MyLab Math.

Top 20 Jobs for Job Satisfaction

1. Mathematician
2. Actuary (works with insurance statistics)
3. Statistician
4. Biologist
5. Software engineer
6. Computer systems analyst
7. Historian
8. Sociologist
9. Industrial designer
10. Accountant
11. Economist
12. Philosopher
13. Physicist
14. Parole officer
15. Meteorologist
16. Medical laboratory technician
17. Paralegal assistant
18. Computer programmer
19. Motion picture editor
20. Astronomer

Source: JobsRated.com.

Everyone wants to find a career path that will bring lifelong job satisfaction, but what careers are most likely to do that? A recent survey evaluated 200 different jobs according to five criteria: salary, long-term employment outlook, work environment, physical demands, and stress. The table to the left shows the top 20 jobs according to this survey. Notice that most of the top 20 jobs require mathematical skills, and all of them require an ability to reason with quantitative information.

You and your classmates can conduct your own smaller study of job satisfaction. There are many ways to do this, but here is one procedure you might try:

- 1 Each of you should identify at least three people with full-time jobs to interview briefly. You may choose parents, friends, acquaintances, or just someone whose job interests you.
- 2 Identify an appropriate job category for each interviewee (similar to the categories in the table to the left). Ask each interviewee to rate his or her job on a scale of 1 (worst) to 5 (best) on each of the five criteria: salary, long-term employment outlook, work environment, physical demands, and stress. You can then add the ratings for the five criteria to come up with a total job satisfaction rating for each job.
- 3 Working together as a class, compile the data to rank all the jobs. Show the final results in a table that ranks the jobs in order of job satisfaction.
- 4 Discuss the results. Are they consistent with the survey results shown in the table? Do they surprise you in any way? Will they have any effect on your own career plans?

What Is Quantitative Reasoning?

Literacy is the ability to read and write, and it comes in varying degrees. Some people can recognize only a few words and write only their names; others read and write in many languages. A primary goal of our educational system is to provide citizens with a level of literacy sufficient to read, write, and reason about the important issues of our time.

Today, the abilities to interpret and reason with **quantitative information**—information that involves mathematical ideas or numbers—are crucial aspects of literacy. These abilities, often called **quantitative reasoning** or **quantitative literacy**, are essential to understanding issues that appear in the news every day. The purpose of this textbook is to help you gain skills in quantitative reasoning as it applies to issues you will encounter in

- your subsequent coursework,
- your career, and
- your daily life.

Quantitative Reasoning and Culture

Quantitative reasoning enriches the appreciation of both ancient and modern culture. The historical record shows that nearly all cultures devoted substantial energy to mathematics and to science (or to observational studies that predated modern science). Without a sense of how quantitative concepts are used in art, architecture, and science, you cannot fully appreciate the incredible achievements of the Mayans in Central America, the builders of the great city of Zimbabwe in Africa, the ancient Egyptians and Greeks, the early Polynesian sailors, and many others.

Similarly, quantitative concepts can help you understand and appreciate the works of the great artists. Mathematical concepts play a major role in everything from the work of Renaissance artists like Leonardo da Vinci and Michelangelo to the pop culture of television shows like *The Big Bang Theory*. Other ties between mathematics and the arts can be found in both modern and classical music, as well as in the digital production of music. Indeed, it is hard to find popular works of art, film, or literature that do not rely on mathematics in some way.

Mathematics knows no races or geographic boundaries; for mathematics, the cultural world is one country.

—David Hilbert (1862–1943),
German mathematician

Quantitative Reasoning in the Work Force

Quantitative reasoning is important in the work force. A lack of quantitative skills puts many of the most challenging and highest-paying jobs out of reach. Table P.1 defines skill levels in language and mathematics on a scale of 1 to 6, and Table P.2 (on the next page) shows the typical levels needed in many jobs.

Note that the occupations requiring high skill levels are generally the most prestigious and highest paying. Note also that most of those occupations call for high skill levels in *both* language and math, refuting the myth that if you're good at language, you don't have to be good at mathematics, and vice versa.

TABLE P.1 Skill Levels

Level	Language Skills	Math Skills
1	Reads signs and basic news reports; writes and speaks simple sentences	Addition and subtraction; simple calculations with money, volume, length, and weight
2	Can read short stories and instruction manuals; writes compound sentences with proper grammar and punctuation	Arithmetic; can compute ratios, rates, and percentages; can draw and interpret bar graphs
3	Reads novels and magazines; writes reports with proper format; speaks well before an audience	Basic geometry and algebra; can calculate discounts, interest, profit and loss
4	Reads novels, poems, and newspapers; prepares business letters, summaries, and reports; participates in panel discussions and debates	Has true quantitative reasoning abilities: understands logic, problem solving, ideas of statistics and probability, and modeling
5	Reads literature, scientific and technical journals, financial reports, and legal documents; can write editorials, speeches, and critiques	Calculus and statistics
6	Same types of skills as level 5, but more advanced	Advanced calculus, modern algebra, and advanced statistics

Source: Adapted from levels described in the *Wall Street Journal*.

TABLE P.2 Skill-Level Requirements

Occupation	Language Level	Math Level	Occupation	Language Level	Math Level
Biochemist	6	6	Web page designer	5	4
Computer engineer	6	6	Corporate executive	5	5
Mathematician	6	6	Computer sales agent	4	4
Cardiologist	6	5	Athlete's agent	4	4
Social psychologist	6	5	Management trainee	4	4
Lawyer	6	4	Insurance sales agent	4	4
Tax attorney	6	4	Retail store manager	4	4
Newspaper editor	6	4	Cement mason	3	3
Accountant	5	5	Poultry farmer	3	3
Personnel manager	5	4	Tile setter	3	3
Corporate president	5	5	Travel agent	3	3
Weather forecaster	5	5	Janitor	3	2
Secondary teacher	5	5	Short-order cook	3	2
Elementary teacher	5	4	Assembly-line worker	2	2
Financial analyst	5	5	Toll collector	2	2
Journalist	5	4	Laundry worker	1	1

Source: Data from the *Wall Street Journal*.

Misconceptions About Mathematics

Do you consider yourself to have “math phobia” (fear of mathematics) or “math loathing” (dislike of mathematics)? We hope not—but if you do, you aren’t alone. Many adults harbor fear or loathing of mathematics, and unfortunately, these attitudes are often reinforced by classes that present mathematics as an obscure and sterile subject.

In reality, mathematics is not nearly so dry as it sometimes seems in school. Indeed, attitudes toward mathematics often are directed not at what mathematics really is but at some common misconceptions about mathematics. Let’s investigate a few of these misconceptions and the reality behind them.

Misconception One: Math Requires a Special Brain

One of the most pervasive misconceptions is that some people just aren’t good at mathematics because learning mathematics requires special or rare abilities. The reality is that nearly everyone can do mathematics. All it takes is self-confidence and hard work—the same qualities needed to learn to read, to master a musical instrument, or to become skilled at a sport. Indeed, the belief that mathematics requires special talent found in a few elite people is peculiar to the United States. In most other countries, particularly in Europe and Asia, *all* students are expected to become proficient in mathematics.

Of course, different people learn mathematics at different rates and in different ways. For example, some people learn by concentrating on concrete problems, others by thinking visually, and still others by thinking abstractly. No matter what type of thinking style you prefer, you can succeed in mathematics.

We are all mathematicians ... [your] forte lies in navigating the complexities of social networks, weighing passions against histories, calculating reactions, and generally managing a system of information that, when all laid out, would boggle a computer.

—A. K. Dewdney, *200% of Nothing*

Misconception Two: The Math in Modern Issues Is Too Complex

Some people claim that the advanced mathematical concepts underlying many modern issues are too complex for the average person to understand. It is true that only a few people receive the training needed to work with or discover advanced mathematical concepts. However, most people are capable of understanding enough about the mathematical basis of important issues to develop informed and reasoned opinions.

The situation is similar for other fields. For example, years of study and practice are required to become a proficient professional writer, but most people can read a book. It takes hard work and a law degree to become a lawyer, but most people can understand how the law affects them. And though few have the musical talent of Mozart, anyone can learn to appreciate his music. Mathematics is no different. If you've made it this far in school, you can understand enough mathematics to succeed as an individual and a concerned citizen.

Skills are to mathematics what scales are to music or spelling is to writing. The objective of learning is to write, to play music, or to solve problems—not just to master skills.

—from *Everybody Counts*,
a report of the National Research
Council

Misconception Three: Math Makes You Less Sensitive

Some people believe that learning mathematics will somehow make them less sensitive to the romantic and aesthetic aspects of life. In fact, understanding the mathematics that explains the colors of a sunset or the geometric beauty in a work of art can only enhance our aesthetic appreciation of these things. Furthermore, many people find beauty and elegance in mathematics itself. It's no accident that people trained in mathematics have made important contributions to art, music, and many other fields.

It is impossible to be a mathematician without being a poet in the soul.

—Sophia Kovalevskaya (1850–1891),
Russian mathematician

Misconception Four: Math Makes No Allowance for Creativity

The “turn the crank” nature of the problems in many textbooks may give the impression that mathematics stifles creativity. Some of the facts, formalisms, and skills required for mathematical proficiency are fairly cut and dried, but using these mathematical tools takes creativity. Consider designing and building a home. The task of construction requires specific skills to lay the foundation, frame in the structure, install plumbing and wiring, and paint walls. But the full process involves much more: Creativity is needed to develop the architectural design, respond to on-the-spot problems during construction, and factor in constraints based on budgets and building codes. The mathematical skills you've learned in school are like the skills of carpentry or plumbing. Applying mathematics is like the creative process of building a home.

Tell me, and I will forget. Show me, and I may remember. Involve me, and I will understand.

—Confucius (c. 551–479 B.C.E.)

Misconception Five: Math Provides Exact Answers

A mathematical formula will yield a specific result, and in school that result may be marked right or wrong. But when you use mathematics in real-life situations, answers are never so clear cut. For example:

A bank offers simple interest of 3%, paid at the end of 1 year (that is, after 1 year the bank pays you 3% of your account balance). If you deposit \$1000 today and make no further deposits or withdrawals, how much will you have in your account after 1 year?

A straight mathematical calculation seems simple enough: 3% of \$1000 is \$30; so you should have \$1030 at the end of a year. But will you? How will your balance be affected by service charges or taxes on interest earned? What if the bank fails? What if the bank is located in a country in which the currency collapses during the year? Choosing a bank in which to invest your money is a *real* mathematics problem that doesn't necessarily have a simple or definitive solution.

Probably the most harmful misconception is that mathematics is essentially a matter of computation. Believing this is roughly equivalent to believing that writing essays is the same as typing them.

—John Allen Paulos, mathematician



People Who Studied Mathematics

The critical thinking skills developed through the study of mathematics are valuable in many careers. The following list represents a small sample of people who studied mathematics but are more famous for work in other fields. (Many of the names come from "Famous Nonmathematicians," a list compiled by Steven G. Buyske, Rutgers University.)

Ralph Abernathy, civil rights leader, BS in mathematics, Alabama State University

Tammy Baldwin, U.S. Senator (Wisconsin), BA in mathematics, Smith College

Sergey Brin, co-founder of Google, BA in mathematics, University of Maryland

Mayim Bialik, actress on *The Big Bang Theory*, studied mathematics in working toward her PhD in neuroscience

Harry Blackmun, former Supreme Court justice, summa cum laude in mathematics, Harvard University

James Cameron, film director, studied physics before leaving college, works in oceanic and space research

Lewis Carroll (Charles Dodgson), mathematician and author of *Alice in Wonderland*

Felicia Day, actress, BA in mathematics, University of Texas

David Dinkins, former mayor of New York City, BA in mathematics, Howard University

Alberto Fujimori, former president of Peru, MS in mathematics, University of Wisconsin

Art Garfunkel, musician, MA in mathematics, Columbia University

Reed Hastings, founder and CEO of Netflix, BA in mathematics, Bowdoin College

Grace Hopper, computer pioneer and first woman rear admiral in the U.S. Navy, PhD in mathematics, Yale University

Mae Jemison, first African-American woman in space, studied mathematics in working toward her BS in chemical engineering, Stanford University

John Maynard Keynes, economist, MA in mathematics, Cambridge University

Hedy Lamarr, actress, invented and patented the mathematical technique called "frequency hopping"

Lee Hsien Loong, Prime Minister of Singapore, BA in mathematics, Cambridge University

Brian May, lead guitarist for the band Queen, completed his PhD in astrophysics in 2007, Imperial College

Danica McKellar, actress, BA with highest honors in mathematics, UCLA, and co-discoverer of the Chayes-McKellar-Winn theorem

Andrea Merkel, Chancellor of Germany, studied mathematics in working toward her PhD in physics, University of Leipzig

Harvey Milk, politician and activist for gay rights, BA in mathematics, State University of New York

Edwin Moses, three-time Olympic champion in the 400-meter hurdles, studied mathematics in working toward his BS in physics, Morehouse College

Florence Nightingale, pioneer in nursing, studied mathematics and applied it to her work

Natalie Portman, Oscar-winning actress, semifinalist in Intel Science Talent Search and co-author of two published scientific papers

Sally Ride, first American woman in space, studied mathematics in working toward her PhD in physics, Stanford University

David Robinson, basketball star, BA in mathematics, U.S. Naval Academy

Alexander Solzhenitsyn, Nobel prize-winning Russian author, degrees in mathematics and physics, University of Rostov

Bram Stoker, author of *Dracula*, BA in mathematics, Trinity University, Dublin

Laurence Tribe, Harvard law professor, summa cum laude in mathematics, Harvard University

John Urschel, NFL offensive lineman (Baltimore Ravens) who retired at age 26 to pursue a PhD in mathematics, MIT

Virginia Wade, Wimbledon champion, BA in mathematics, Sussex University

Misconception Six: Math Is Irrelevant to My Life

No matter what your path in college, career, and life, you will find mathematics involved in many ways. A major goal of this text is to show you hundreds of examples in which mathematics applies to everyone's life. We hope you will find that mathematics is not only relevant but also interesting and enjoyable.

Neglect of mathematics works injury to all knowledge . . .

—Roger Bacon (1214–1294), English philosopher

Misconception Seven: It's OK to Be “Bad at Math”

For our final misconception, let's return to the multiple-choice question on the opening page of this Prologue. You'll not only hear many otherwise intelligent people say “I'm bad at math,” but it's sometimes said almost as a point of pride, with no hint of embarrassment. Yet the statement often isn't even true. A successful lawyer, for example, almost certainly did well in *all* subjects in school, including math, and so is more likely expressing an attitude than a reality.

You must be the change you wish to see in the world.

—Mahatma Gandhi (1869–1948)

Unfortunately, this type of attitude can cause a lot of damage. Mathematics underlies nearly everything in modern society, from the daily financial decisions that all of us must make to the way in which we understand and approach global issues involving the economy, politics, and science. We cannot possibly hope to act wisely if we approach mathematical ideas with a poor attitude. Moreover, it's an attitude that can easily spread to others. After all, if a child hears a respected adult saying that he or she is “bad at math,” the child may be less inspired to do well.

So before you begin your coursework, think about your own attitudes toward mathematics. There's no reason why anyone should be “bad at math” and every reason to develop skills of mathematical thinking. With a good attitude and some hard work, by the end of this course you'll not only be better at math, but you'll be helping future generations by making it socially unacceptable for anyone to be “bad at math.”

What Is Mathematics?

In discussing misconceptions, we identified what mathematics is *not*. Now let's look at what mathematics *is*. The word *mathematics* is derived from the Greek word *mathematikos*, which means “inclined to learn.” Literally speaking, to be mathematical is to be curious, open-minded, and always interested in learning more! Today, we tend to look at mathematics in three different ways: as the sum of its branches, as a way to model the world, and as a language.

Mathematics as the Sum of Its Branches

As you progressed through school, you probably learned to associate mathematics with some of its branches. Among the better-known branches of mathematics are these:

- **logic**—the study of principles of reasoning;
- **arithmetic**—methods for operating on numbers;
- **algebra**—methods for working with unknown quantities;
- **geometry**—the study of size and shape;

- **trigonometry**—the study of triangles and their uses;
- **probability**—the study of chance;
- **statistics**—methods for analyzing data; and
- **calculus**—the study of quantities that change.

One can view mathematics as the sum of its branches, but in this text we'll focus on how different branches of mathematics support the more general goals of quantitative thinking and critical reasoning.

Mathematics as a Way to Model the World

Mathematics also may be viewed as a tool for creating *models*, or representations of real phenomena. Modeling is not unique to mathematics. For example, a road map is a model that represents the roads in some region.

Mathematical models can be as simple as a single equation that predicts how the money in your bank account will grow or as complex as a set of thousands of interrelated equations and parameters used to represent the global climate. By studying models, we gain insight into otherwise unmanageable problems. A global climate model, for example, can help us understand weather systems and ask “what if” questions about how human activity may affect the climate. When a model is used to make a prediction that does *not* come true, that failure points out areas where further research is needed. Today, mathematical modeling is used in nearly every field of study. Figure P.1 shows some of the many disciplines that use mathematical modeling to solve problems.

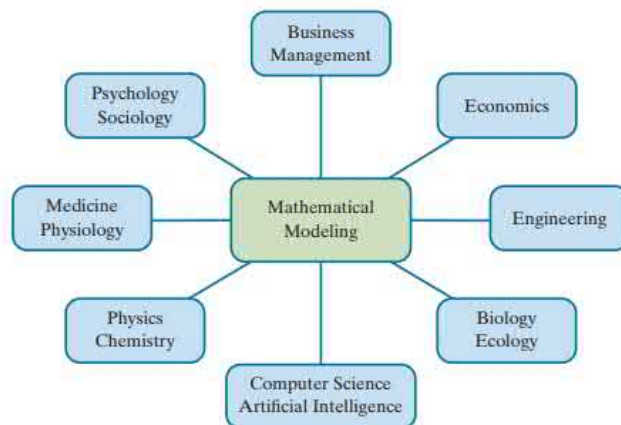


FIGURE P.1

Mathematics as a Language

The Book of Nature is written in the language of mathematics.
—Galileo

A third way to look at mathematics is as a language with its own vocabulary and grammar. Indeed, mathematics often is called “the language of nature” because it is so useful for modeling the natural world. As with any language, different degrees of fluency are possible. From this point of view, quantitative literacy is the level of fluency required for success in today’s world.

The idea of mathematics as a language is also useful in thinking about how to *learn* mathematics. Table P.3 compares learning mathematics to learning a language and learning about art.